Practical Exercise

Compute the Fundamental Matrix from two simulated cameras.

Description: Describe two simulated cameras. Construct their transformation matrices. Get the Fundamental matrix analytically. Define a set of 3D points and get their corresponding couples of projecting points. Compute the Fundamental matrix by using the 8-point method. Compare both fundamental matrices. Draw the epipolar geometry in both images planes (points, epipoles and epipolar lines). Increase the noise in 2D points and repeat the computation. Check the consistency of the epipolar geometry obtained.

Programming platform: Matlab

Part 1.

Step 1. Define camera 1 with the following parameters and set the world coordinate system to the coordinate system of camera 1 (Rotation=Identity and translation=0):

```
au1 = 100; av1 = 120; uo1 = 128; vo1 = 128;
Image size: 256 x 256
```

Step 2. Define camera 2 with respect to camera 1 with the following parameters:

```
au2 = 90; av2 = 110; uo2 = 128; vo2 = 128;
ax = 0.1 rad; by = pi/4 rad; cz = 0.2 rad;   XYZ EULER
tx = -1000 mm; ty = 190 mm; tz = 230 mm;
Image size: 256 x 256
```

Step 3. Get the intrinsic transformation matrices of both cameras, and the rotation and translation between both cameras. Be aware that the dimensions of the intrinsic matrices are 3x3, the rotation matrix is 3x3 and the translation vector is 1x3.

Step 4. Get the Fundamental matrix analytically as the product of matrices defined in step 3, as follows. Be aware to put the translation vector in the form of its antisymmetric matrix (see lecture Rigid Body Transformations).

$$F = \mathbf{A}'^{-t} R^t \left[t\right]_x \mathbf{A}^{-1}$$

Step 5. Define the following set of object points with respect to the world coordinate system (or camera 1 coordinate system)

```
V(:,1) = [100;-400;2000;1];
V(:,2) = [300;-400;3000;1];
V(:,3) = [500;-400;4000;1];
V(:,4) = [700;-400;2000;1];
V(:,5) = [900;-400;3000;1];
V(:,6) = [100;-50;4000;1];
V(:,7) = [300;-50;2000;1];
V(:,8) = [500;-50;3000;1];
V(:,9) = [700;-50;4000;1];
V(:,10) = [900;-50;2000;1];
V(:,11) = [100;50;3000;1];
V(:,12) = [300;50;4000;1];
V(:,13) = [500;50;2000;1];
V(:,14) = [700;50;3000;1];
V(:,15) = [900;50;4000;1];
V(:,16) = [100;400;2000;1];
V(:,17) = [300;400;3000;1];
V(:,18) = [500;400;4000;1];
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V(:,19) = [700;400;2000;1];
V(:,20) = [900;400;3000;1];
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Step 6. Compute the couples of image points in both image planes by using the matrices of step 3.

[Hint] Be aware that now you have to use the corresponding intrinsic matrix with dimensions 3x4 and the corresponding extrinsic matrix with dimensions 4x4, to project the 3D points onto both image planes and get both projections in pixels. Be aware that one of the cameras is located at the origin of the world coordinate system and hence Ext1= ${}^{C}K_{W}$ = [I 0] = ${}^{W}K_{C}$. Be aware that the second camera is located at ${}^{C}K_{C'}$= ${}^{W}K_{C'}$ = [R t] and hence Ext2= ${}^{C'}K_{W}$ =[R^t –R^tt].

Step 7. Open two windows in matlab, which will be used as both image planes, and draw the 2D points obtained in step 6.

Step 8. Compute the fundamental matrix by using the 8-point method and least-squares by means of the 2D points obtained in step 6.

[Hint] Be aware that the system of equations is in the following form, which differs from the system of equations given in the slides.

$$m'^{t} Fm = 0$$

$$(x', y', 1)F\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

Step 9. Compare the step 8 matrix with the one obtained in step 4.

Step 10. Draw in the windows of step 7 all the epipolar geometry, i.e. epipoles and epipolar lines by using the matrix obtained in step 8. Enlarge the windows if necessary to view the epipoles properly.

[Hint] An epipolar line is defined by l'm'=Fm=$[u1,u2,u3]^T$. Besides any point [x,y,1] that lies on its corresponding epipolar line satisfies [x,y,1] $[u1,u2,u3]^T$=0. From this equation you can extract y=mx+d, so that you obtain m and d from the components of the epipolar line. Hence, fixing x in both boundaries of the image plane, you can obtain the y component and draw the epipolar line by using the Plot function.

[Hint] You can compute the epipoles by projecting the focal point of each camera to the image plane of the other. Moreover, the epipoles can be computed by intersecting two or more epipolar lines as all the epipolar lines cross at the epipole in the absence of noise.

Step 11. Add some Gaussian noise to the 2D points producing discrepancies between the range [-1,+1] pixels for the 95% of points.

Step 12. Again repeat step 8 up to 10 with the noisy 2D points. Compare the epipolar geometry obtained (are points on the epipolar lines?, are the epipoles a unique point?).

[Hint] In this case you may note that epipolar lines do not cross at a single point, i.e. the epipole is not unique, because the Fundamental matrix obtained is rank-3. You can compute the closest rank-2 matrix from any rank-3 matrix by SVD (Use the SVD function). That is, F=UDV ^{T}. The diagonal matrix D is 3x3 and if we set the smallest eigenvalue to zero and calculate F again, we end up with a fundamental matrix of rank 2. Then, you can compute the epipolar lines from the rank-2 matrix and verify if all cross in the epipole.

[Hint] You can also compute the epipoles directly from F. The definition of the epipole in the left image can be given by (pr F el = 0) for all pr. A similar definition exists for the right epipole: (er F pl = 0) for all pl. How can we calculate the epipole then? Well, F is not identical to zero (it has rank 2), so that means that if the above equations must hold, that: (F el = 0) and (er F = 0). In other words, the epipole el must lie in the nullspace of F, and similar er must lie in the null space of F^T If we take the SVD of the fundamental matrix, el is a multiple of the column of V that belongs to the zero singular value of F (remember F is singular and of rank 2). Similar, er will be a multiple of the column of U corresponding to the zero singular value. Note that the epipoles are only known up to a scaling factor.

Step 13. Increase the Gaussian noise of step 11 (now in the range [-2,+2] for the 95% of points) and repeat step 8-12.

Part 2.

Step 14. Compute the fundamental matrix by using the 8-point method and SVD from the points 2D obtained in step 6 and without noise. Compare the obtained matrix with the one obtained in step 8.

[Hint] Any system of equations AX=0 ($X=[F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{32}, F_{33}]^T$) can be solved by SVD so that X lies in the nullspace of A. Hence X corresponds to a multiple of the column of V that belongs to the zero singular value of A. Note that X is only known up to a scaling factor.

Step 15. Repeat step 10 up to 13 (with the matrix of step 14 instead of step 8) for some Gaussian noise first in the rang [-1, 1] and then in the rang [-2, 2] for the 95% of points.

Step 16. Compare the epipolar geometry obtained in step 15 (using SVD) with the one obtained in steps 11 and 13 (using LS). Which of both fundamental matrices minimizes the distance between points and epipolar lines?